PROJECTED WRITTEN NOTES from the M325K LECTURE
ON Tuesday, January 16, 2024 on The M325K CANVAS COURSE,
The class Syllabus and the Relationship "Congruence (mod n)"
and "Defining variables in proofs"
CLASS # 1

The first half of the class was spent covering the resources available through the links in the "Hot Links List" on the Home Page of the CANVAS COURSE.

Handouts, the Syllabus, the Projected Written Notes and Videos of past lectures, Homework assignments, and Homework Solutions
are all available through the HOT LINKS.

So, too, are the CANVAS DAILY PAGES READING Assignments and other information to follow to be ready for the next lecture.

Also discussed were the details in the Syllabus, which everyone should read.

Next, we discussed the information in the handout
"Defining Variables in a Proof"

FINALLY, we discussed the relationship that two integers a and b might have with regard to division by a fixed, positive, integer n, the relationship of "Congruence modulo n"
or "Congruence (mod n)".

The Definition of "(mod n)-congruence"

Definition: Let n be a positive integer.

Suppose a and b are any integers.

We say "a is congruent to b modulo n"
(and we write "$a \equiv b \pmod{n}$")

if and only if $a - b$ is an integer multiple of n.

That is, $a \equiv b \pmod{n} \iff a - b = nk$ for some integer k.

Note: If $a - b = nk$, then $b - a = n \times (-k)$, so
$a \equiv b \pmod{n} \iff b \equiv a \pmod{n}$.

For example, $19 \equiv 7 \pmod{3}$, since $19 - 7 = 12$ and $12 = 3 \times 4$.
Also, $19 \not\equiv 8 \pmod{3}$, since $19 - 8 = 11$ and $11 \neq 3k$ for every integer k.

Theorem: For every positive integer n and every positive integer a,
if r is the remainder when a is divided by n, then
$a \equiv r \pmod{n}$.

Proof: Let n be any positive integer and let a be any positive integer. When a is divided by n, this division results in a quotient q and a remainder r.

$$\begin{array}{r|l} n & a \\ & \underline{-qn} \\ & r \end{array} \qquad \left[\begin{array}{r} q \\ n\overline{)a} \\ \underline{-qn} \\ r \end{array}\right]$$

Thus $a = nq + r$ and $0 \leq r < n$.
$\therefore a - r = nq$ and q is an integer.
$\therefore a \equiv r \pmod{n}$. QED, by Direct Proof.

Let $n=3$, $19-7=12=4\times 3$

$k \cdot n$

$19 \equiv 7 \pmod 3$

$20 \not\equiv 7 \pmod 3$